Subtleties Associated With Derived Demand Relationships

Ronald A. Schrimper

Subtleties involving measurement of quantities and prices when derived demands are graphically displayed in frameworks representing market linkages are discussed. Complications arising from assuming variable proportions rather than fixed coefficients are noted. Finally, an example developed by Wohlgenant and Haidacher is clarified.

Derived demand and supply concepts are extremely useful for illustrating market linkages and analyzing simultaneous market equilibra at the farm and retail levels. Incorporating derived demand and supply relationships into an overall framework provides a useful way for considering agricultural marketing activities connecting production and consumption behavior. The cost of all marketing activities represented by the vertical distance between retail demand and derived demand (or between farm supply and derived supply) in the same diagram is a convenient way for illustrating and thinking about differences between retail and farm prices.[1]

Recent work by Wohlgenant and Haidacher and others has provided additional insight about linkages between retail and farm level demands for agricultural products. Many subtleties however are encountered when attempting to carefully explain key elements about market linkage to students or illustrate the basic framework in the context of a particular problem. The first section of this paper discusses some of the subtleties associated with graphically depicting the basic framework.[2] The second part of the paper clarifies one of the examples used by Wohlgenant and Haidacher in illustrating the difference between fixed and variable proportions of farm commodities used for producing retail food products and describes some of the complications associated with graphically demonstrating the effects of variable proportions.

Basic Framework and Simplifying Assumptions

For initial graphical presentations of derived demand (and/or supply) concepts, it is easiest to assume that the marketing system operates with fixed coefficients of production in converting primary commodities into goods purchased by consumers.[3] It is important to note, however, that this assumption is an oversimplification that can be relaxed after familiarity with the basic ideas is mastered. A major component of Wohlgenant and Haidacher's model as well as Gardner's earlier work is that the price elasticity of derived demand for farm products depends on the elasticity of substitution among inputs in the production of retail food products similar to Hick's conclusion regarding demands for factors of production. Gardner did not include any graphical illustrations in his article but noted that under the assumption of fixed proportions in food marketing, the relationships can be derived by graphical methods like those of Tomek and Robinson (in the first or subsequent editions). Unfortunately, familiarity with graphical results for fixed proportions can lead to an erroneous impression that all derived demands and/or supplies are linked to primary behavioral relationships in a very simple way depending on whether marketing costs per unit are fixed or vary with volume moving through the marketing system.

One of the subtleties encountered in illustrating the market linkages in a single diagram is the selection of a standardized set of measurements for

Professor, Department of Agricultural and Resource Economics, North Carolina State University. The author acknowledges the helpful comments of two anonymous reviewers.

[1] Vertical distances between farm level supply and retail demand (plotted in the same diagram) are used by Fisher to map a demand for marketing services.

[2] No algebraic representation of the basic relationships is introduced in this manuscript as recommended by a reviewer because it would be repetitive of Wohlgenant and Haidacher's presentation.

[3] Often for short run analysis and perishable commodities, it is appropriate to consider the supply at the farm level to be perfectly inelastic. In these cases, the derived supply at the retail level would also be perfectly inelastic and the difference between farm and retail prices can be illustrated entirely by the relationship between primary and derived demand.

expressing quantities as well as prices at different levels of the marketing system. Specification of units is required for appropriate vertical differences between demands (or supplies) at different levels of the marketing system, to be economically meaningful in terms of representing the cost of market linkage services *per unit of product*.[4] For example, in order for vertical distances between the retail and farm demand relationships for beef to represent the marketing costs per lb. of *liveweight*, it is necessary for the retail demand as well as the derived demand to be expressed in equivalent *liveweight units*. Of course, an equally valid alternative representation of the relationships would be to express both demand relationships in terms of retail weight equivalents. In the latter case, vertical distances would represent marketing costs per unit of retail weight rather than per unit of liveweight. This means that starting with point A (or any other arbitrary point on D_r) in Figure 1, the appropriate positioning of point B on the derived demand curve can be determined, provided the share of the final retail price accounted for by marketing costs is specified.

An alternative representation of Figure 1 in terms of units of raw products under fixed conversion coefficients could be illustrated by rescaling the vertical and horizontal axes by the appropriate conversion factor. For example, a farm price of \$2.40/per lb. of retail weight can easily be converted to an equivalent price of raw product or farm weight provided the appropriate factor for converting quantities at one level into quantities at the other level is known. If 2.4 lbs. of raw product are required to produce one lb. of retail weight, point B on the derived demand is equivalent to a farm price of \$1/per lb. of raw product (or farm weight) if prices for the primary and the derived demand are expressed per unit of liveweight in Figure 1.[5] Points A and B would be vertically aligned at an alternative quantity value equal to $Q/2.4$ with P_r and P_f being similarly deflated in terms of values per unit of liveweight.

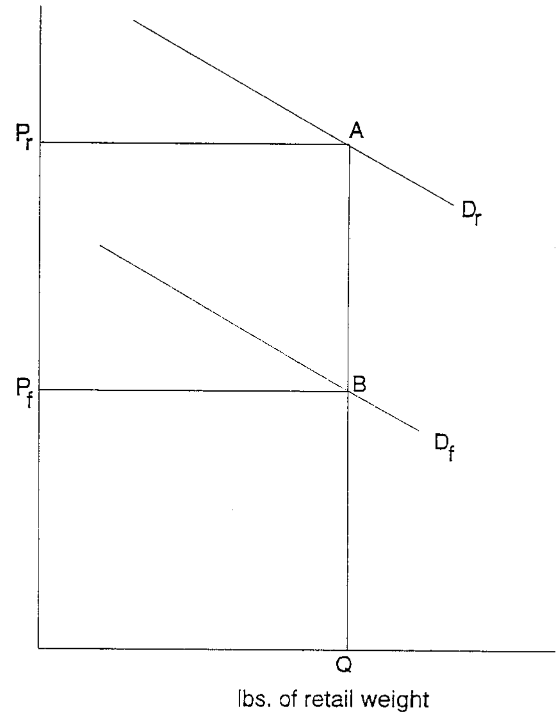

Figure 1. Primary and derived demand relationships with fixed coefficients and perfectly elastic supply of marketing inputs.

When introducing the concept of derived demand to students, complications associated with quantity conversions (as well as product aggregation) can be largely avoided by selecting watermelons for illustration purposes and noting that the quantities of raw product and retail product are essentially the same.[6] This finesses having to be overly concerned about the appropriate units for the horizontal axis. Graphical and/or algebraic representations of the different relationships can be used without being specific about quantity and price units by noting the relationships can be made compatible as long as one knows how to convert price and quantity combinations at one level of the

[4] The quantity dimension used for expressing prices per unit on the vertical axis of such diagrams frequently is identical to that selected for the horizontal axis, but does not have to be. For example, the horizontal axis could be expressed in tons, but prices might be dollars per lb. or expressed in terms of some other quantity unit. The critical issue for such diagrams is that prices at various levels of the marketing system be comparable in order for vertical distances for particular quantities to be economically meaningful. Gardner's article examines price spreads and relative price ratios that are economically meaningful if quantity units are identical or remain in a fixed proportion.

[5] Actually, only the vertical axes would need to be converted to change the vertical representation of marketing costs in terms of raw product rather than retail product. Alternative retail and farm prices per unit of raw product could be plotted for alternative quantities of retail product to represent the kind of demand relationships in Figure 1.

[6] Even though watermelons are sold at the retail level often as cutup products, it is fairly easy to think of the total quantity (measured either in pounds or total number) of watermelons sold at the retail level to be essentially the same quantity produced and sold at the farm level. Adjustments for shrink, spoilage and other quantity losses as well as adjustments for the value of by-products resulting from the marketing process need to be acknowledged and must be incorporated as part of the difference between retail and farm prices for any particular total quantity of farm or retail product sold. Another subtlety involved in linking retail and farm level demands is determining a correspondence between farm and retail products. Considerable aggregation of derived demands associated with different retail products may be required to consider the aggregate demand for any particular agricultural product used in the production of a variety of retail products.

marketing system into comparable values at other stages.

Fixed vs. Variable Proportions

Assuming fixed coefficients of production for converting primary agricultural products into retail food products greatly simplifies graphical representations of market linkages. The extent to which changes in prices of agricultural products relative to other inputs used by marketing firms however affect the incentive to alter the combination of agricultural products and other inputs in the short run as well as long run implies that fixed coefficiences may be an oversimplifying assumption. Wohlgenant and Haidacher present a strong case for considering the possibility of variable proportions in considering market linkages. The diagram they use to illustrate the effect of assuming variable proportions instead of fixed coefficients on the price elasticity of derived demand initially looks very much like what occurs under fixed coefficients of production if marketing costs per unit of product decrease with increasing quantities.

Although introducing variable proportions produces what appears to be the same type of graphical representation of retail and derived demands as when marketing costs per unit decrease, the interpretation of the relationships is much more complicated. For example, the diagram used by Wohlgenant and Haidacher to illustrate the economic implications of this change in assumptions, has the quantity axis initially specified in terms of units (actually lbs.) of retail product. The price axis represents retail and farm price per unit of retail weight with vertical distances representing marketing cost per unit of retail product (i.e., $P_r - P_f$). Thus, the difference between P_r^0 and P_f^0 in Figure 2 is assumed to represent the marketing cost per unit of retail product under either fixed or variable proportions when 1,000 units of retail products (or 2,400 corresponding units of farm products) move through the marketing system. Similarly the difference between P_r^1 and P_f^1 would represent the marketing cost per unit of retail product under fixed coefficients of production when 500 units of retail product (or 1,200 units of farm products) move through the marketing system. In the case of a fixed conversion factor, the derived demand is a direct vertical descendant of the retail demand curve. Each point on the derived demand curve represents the farm price that is consistent with markets clearing for a given quantity (measured either in terms of retail or live weight) provided by producers and purchased by consumers, ignoring

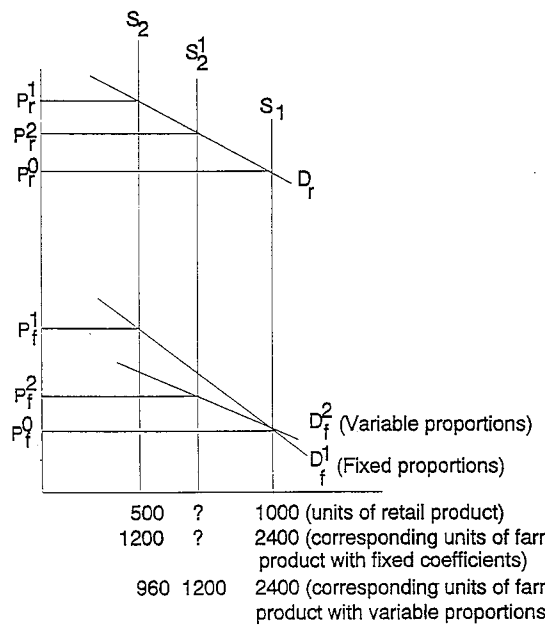

Figure 2. Wohlgenant and Haidacher example.

any intermediate storage or leakage in the marketing channels.[7] With variable proportions however, it is necessary to incorporate adjustments in quantity conversion factors to determine appropriate vertical positioning of points on the derived demand curve as different points on the retail demand curve are considered. This is required in order for the vertical distances between the two demand functions to be interpreted as the cost of marketing per unit of the retail product. For example, under variable proportions when a reduction in farm supply is accompanied by a change from 2.4 lbs. to 1.92 lbs. of raw product per unit of retail, P_r^1 would be observed only if the reduction in farm product were from 2,400 to 960 rather than from 2,400 to 1,200 units. This means that a given movement along the retail demand function (i.e., from P_r^0 to P_r^1) can result from two entirely different changes in supply at the farm level depending on whether product conversion occurs in fixed or variable proportions.

The implicit optimization process involved in combining raw products with marketing inputs makes the graphical representation of the derived

[7] The difference between P_r^1 and P_f^1 could be the same or vary from the difference between P_r^0 and P_f^0 depending on whether the price (or cost) of marketing inputs varies with quantity of the product moving through the marketing system (i.e. whether the price elasticity of the supply function of marketing inputs is something other than perfectly elastic).

demand for the raw product on the same diagram with retail demand under variable proportions more complex than in the case of fixed proportions. The derived demand function representing the relationship between P_f and Q_f (expressed in retail weight equivalent units) depends on the nature of retail demand, the supply of marketing inputs and technological substitution possibilities. This means each point along the D_f^2 represents an equilibrium farm price for a specific quantity of raw product, conditional on a particular level of retail demand (expressed in retail weight units), supply of marketing inputs and potential substitution possibilities in producing retail products. Comparing points on derived demand curves with corresponding market clearing equilibrium values on retail demand functions can be tricky.

Wohlgenant and Haidacher's discussion of the relationships contained in Figure 2 is a little confusing in that they refer to their diagram as illustrating the different effects of a *given* shift to the left of a perfectly inelastic supply of the farm product under fixed or variable proportions. Unfortunately the numbers and points they selected in the diagram involve the effects of *two different shifts* of a perfectly inelastic farm supply. In the one case, farm supply is assumed to decrease by 50% (from 2,400 to 1,200 of corresponding units of farm product under fixed proportions) but by 60% (from 2,400 to 960 of corresponding units of farm product under variable proportions). Each of these changes produces the same decrease in retail supply of 50 percent as other inputs are used in place of some of the farm product if substitution is feasible.

In order to compare the effects of a *specific* decrease (say 50 percent) in farm supply under fixed vs. variable proportions, two different changes along the retail demand function would be required. Under variable proportions, a retail price lower than P_r^1 would occur, (for example, perhaps P_r^2) corresponding to a retail quantity somewhat greater than 500. This is the result of a smaller percentage reduction in retail quantities being associated with a given decrease in units of the raw product under variable proportions compared to fixed proportions. Subtracting the cost of marketing inputs per unit of retail product from P_r^2 produces the appropriate net price per unit of retail product that marketing firms would be willing to pay for the raw product after a 50 percent reduction in farm supply under variable proportions. A 50 percent reduction in farm supply might be consistent with only a 40 percent reduction (S_1 to S_2^1) in retail supply under variable proportions instead of the 50 percent reduction that would occur with fixed coefficients of production (S_1 to S_2). This illustrates why it is important to remember that graphing primary and derived demand relationships on one diagram involves using the same units along the horizontal (as well as the vertical) axis for both relationships.

Alternative Graphical Representation

Another way to illustrate the effects of a specific change in farm supply would be to consider the retail and farm demand functions on separate diagrams using different units for the horizontal axes as in Figure 3. For the retail demand, the quantity axis would be in terms of units of retail product. The quantity axis for the derived demand could be expressed in terms of units of farm product to illustrate the effects of a *specified* reduction in farm supply under fixed or variable coefficients. The price for both diagrams could be expressed as $ per lb. of retail weight equivalent or $ per lb. of raw product equivalent in order for the difference in equilibrium prices to be economically meaningful as a measure of marketing costs for alternative market equilibria. For consistency with the earlier discussion, it is easiest to consider expressing prices at each level of the marketing system (and marketing costs) per unit of retail product.[8]

Assuming a marketing cost of $1.60 per lb. of retail product and a constant conversion factor of 2.4 lbs. of raw product per lb. of retail product, each of the points on the retail demand function can be converted into a equivalent farm level price for each quantity combination similar to what was discussed earlier. For example, a retail price of $4.00/lb. for 1,000 units of retail product would be consistent with a price of $2.40/lb. (retail weight) for 2,400 units of raw product on the derived demand function. Similarly, if the market clearing retail price for 500 units of retail product were $6.00 per lb., a corresponding point on the derived demand function would be $4.40 per lb. for 1,200 units of raw product. Similarly a linear specification of the retail demand function would imply a retail price of $5.50 per lb. and a farm price of $3.90 per lb. for 625 lbs. of retail product and 1,500 lbs. of raw product under a fixed conversion factor of 2.4.

Assuming that the conversion factor changes rather than remains constant when the supply of raw product decreases, results in an entirely dif-

[8] If prices and marketing costs are expressed in terms of units of raw product, the effect of variable proportions requires a translation of the retail demand instead of the derived demand function.

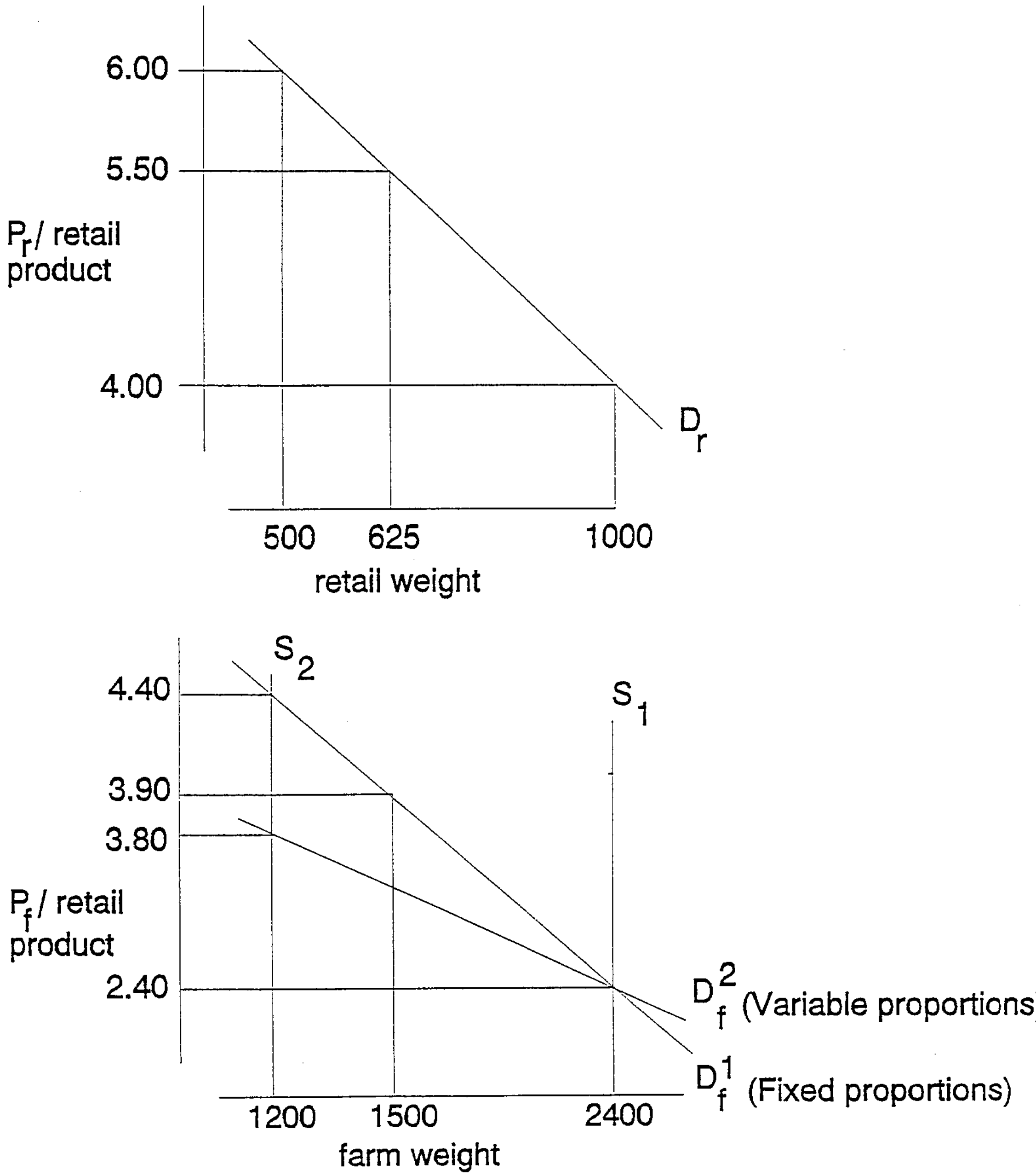

Figure 3. Retail and derived demands with different horizontal axes.

ferent derived demand function associated with the same retail demand specification. As noted earlier a decrease from 2,400 to 1,200 units of raw product would produce a change from 1,000 to 625 units of retail product if the conversion factor is reduced from 2.4 to 1.92. The substitution of additional marketing inputs in place of some of the raw product used to produce retail products would be expected to increase marketing costs per unit of retail product even if the price of marketing inputs remain unchanged (i.e., a perfectly elastic supply of marketing inputs). Assuming no change in retail demand implies that consumers would be willing to pay the same price per unit for identical quantities of retail product under either set of circumstances. For example, if consumers are willing to pay $5.50 per unit of retail product when 625 units are available regardless of whether the conversion factor is 2.4 or 1.92, implies that the derived demand price would be less than $3.90 (e.g., perhaps $3.80) for 1,200 units of raw product under variable coefficients. Considering the derivation of similar price valuations for other alternative equilibrium combinations of quantities along the two quantity axes leads to the development of a derived demand function for the raw product under variable proportions with a different slope than under fixed coefficients consistent with the theoretical model of Wohlgenant and Haidacher.

Each of the derived demand functions in Figure 3 could be represented in a more conventional diagram with the price axis expressed in $ per unit of farm product assuming the appropriate transformation coefficient is known for each aggregate quantity under fixed or variable proportions. In the case of fixed coefficients, the translation would be very straightforward. For example, the points along the derived demand function with fixed coefficients associated with 2,400 and 1,500 units of raw product could be equally represented in terms of prices of $1.00 and $1.625 per unit of raw product (i.e., $2.40/2.4 and $3.90/2.4). This implies that it wouldn't matter whether proportional price differences were compared using either measure for the vertical axes (e.g., a 62.5% increase in price).

In the case of variable coefficients, the translation process would be more complicated and a significance difference could occur depending on whether farm level prices per unit of retail product or per unit of farm product along derived demand functions are compared. For example, comparing the percentage change in price in Figure 3 along the derived demand function associated with a reduction of 50 percent in quantity supplied implies an increase of 58.3 percent (from $2.40 to $3.80) using farm prices per unit of retail product. Incorporating a change in conversion coefficients from 2.4 to 1.92 and using farm prices per unit of raw product implies a 97.9% (from $1.00 to $1.979) increase in price at the farm level associated with a 50 percent reduction in quantity supplied at the farm level.

Summary

The paper discusses some subtleties associated with measurement of quantities and prices at different levels of the marketing system that are encountered in frameworks used to represent market linkages. Attention is focused on some of the complications that arise in illustrating market linkages when the simplifying assumption of combining basic agricultural products and marketing services in fixed proportions is modified to consider the more realistic possibility of variable proportions. At first glance a graphical depiction of the latter situation appears to be similar to what occurs in the case of a downward sloping supply of marketing services under the fixed proportion assumption. Under variable proportions multiple market equilibrium have to be interpreted carefully in order for vertical distances between primary and derived demand curves to reflect appropriate marketing costs per unit of product. The most important interpretation of the relationships depicted in Figure 2 is the same as stated by Wohlgenant and Haidacher. That is under variable proportions, marketing costs per *unit of retail product* tend to be inversely related to quantities moving through the marketing system even if marketing inputs have perfectly elastic supplies. A minor qualification is to note that two changes along the retail demand function must be considered rather than one (as assumed by Wohlgenant and Haidacher) to illustrate the differential effects on market equilibra and marketing costs resulting from a *given* change in a perfectly inelastic supply of agricultural products under variable proportions relative to fixed proportions.

References

Fisher, B.S. "The Impact of Changing Marketing Margins on Farm Prices," *American Journal of Agricultural Economics*, 63, No. 2(May 1981):261–263.

Gardner, Bruce L. "The Farm-Retail Price Spread in a Competitive Food Industry," *American Journal of Agricultural Economics*, 57, No. 3(August 1975):399–409.

Hicks, J.R. *The Theory of Wages*, St. Martin's Press, New York, 1963.

Tomek, W.G. and K.L. Robinson. *Agricultural Product Prices*. Ithaca, N.Y., Cornell University Press, 1972.

Wohlgenant, Michael K. and Richard C. Haidacher. *Retail to Farm Linkage for a Complete System of Food Commodities*. Technical Bulletin 1775, Economic Research Service, USDA, Washington, D.C., December 1989.